Week 3 PSO Questions

Mukul Agarwal (TA)

September 8, 2026

Recursion Practice. Solve each recurrence to a tight asymptotic bound using a recursion tree. Assume $T(O(1)) = O(1)$.

1. $T(n) = T(n/2) + T(n/3) + T(n/7) + n \log n$.
2. $T(n) = 2T(n/4) + T(n/2) + n \log n$.

One Free Absence. A student records how much they learned on consecutive study days. The result is an array $A[1..n]$ of positive and negative numbers. They want to choose a nonempty contiguous interval of days with the largest total score, but they may discard the score from at most one day within the chosen interval.

For example, in $(4, -10, 5, 3)$, choosing all four days and discarding the second day gives a score of 12.

1. Give a straightforward $O(n^3)$ algorithm for this problem.
2. Give a faster algorithm. The faster, the better.

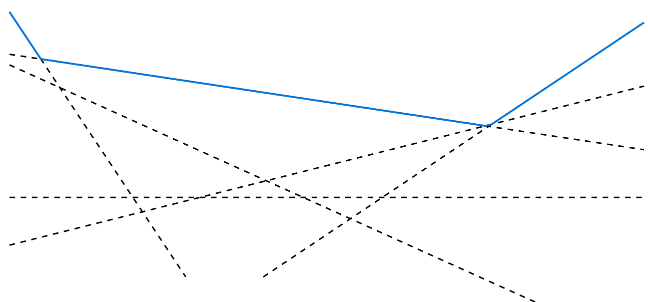
Upper Envelope.[1] Let L be a set of n lines in the plane, where line ℓ_i is given by $y_i(x) = a_i x + b_i$ for $a_i, b_i \in \mathbb{R}$. The **upper envelope** of L is the function

$$U_L(x) = \max_{i \in [n]} y_i(x).$$

The function U_L is piecewise linear and convex. Represent it by a list of **breakpoints**

$$(x_1, \ell_1), (x_2, \ell_2), \dots, (x_m, \ell_m),$$

where $x_1 < x_2 < \dots < x_m$ are the coordinates at which the topmost line changes, and ℓ_i is the line attaining the maximum on $[x_i, x_{i+1})$. Also specify which line is topmost for $x < x_1$.



For example, consider the three lines $\ell_1 : y = -x + 4$, $\ell_2 : y = 2$, and $\ell_3 : y = x$. Their upper envelope is attained by ℓ_1 for $x < 2$ and by ℓ_3 for $x \geq 2$. All three lines tie at $x = 2$; ℓ_2 is never uniquely topmost and does not need its own envelope segment.

Design and analyze a recursive algorithm, as fast as possible, that computes the upper envelope of n lines in the plane.[2]

Remark. *The upper envelope appears in computational geometry (convex hulls, linear programming) and in the "convex hull trick" for optimizing certain dynamic programs.*

[1]This problem is due to Kent Quanrud.

[2]*Hint.* The n lines determine $\binom{n}{2}$ pairwise intersections, but how many breakpoints can their upper envelope have?